Sets (Section 6.1)

Hyperlinks are shown in blue, download the `cdf player` *from the Wolfram Alpha website to view the* `Wolfram Alpha interactive demonstrations`*. When you have downloaded the cdf player, click on this symbol to view the demonstration.*

A **set** is a collection of objects. The objects are called **elements** of the set.

A set can be described as a list, for example $D = \{5, 6, 7\}$ or with words:

$$D = \{\text{All whole numbers between 5 and 7 inclusive}\}$$

Note Repetitions in the list or changes in the order of presentation do not change the set. For example, the following lists describe the same set:

$$\{5, 5, 6, 7\} = \{5, 6, 7\} = \{6, 5, 7\}.$$

When describing the elements of a set, we should be careful that there is **no ambiguity in our description** and the set is well defined. For example to talk about "the "set" of the ten greatest sportsmen or sportswomen of the twentieth century" does not make sense, since the description is subject to personal opinion and different people may produce different sets from the description.

Note however that a verbal description of a well defined set is not necessarily unique. For example the set D above might be described as: $D = \{$ All integers bigger than 4 and less than 8$\}$.

Notation We read the notation $5 \in D$ as "5 is an element of D". The statement $2 \notin D$ is read as "2 is not an element of D".

Equal Sets We say two sets are **equal** if they consist of exactly the same elements. For example consider the following sets
$A = \{$Major league baseball players who got more than 760 home runs in their career $\}$,
$B = \{$Major league baseball players who got more than 70 home runs in a single season$\}$. These two sets are equal and have a single element. We have $A = B = \{$Barry Bonds$\}$.

Infinite Sets and dot notation A set may have infinitely many elements, in which case it is impossible to list all of them. For example let $\mathbf{E} = \{$all even integers greater than or equal to 1$\}$. We cannot list all elements of this set, however we can use a special mathematical notation for *etcetera* to describe the list of elements. We write this list as $\mathbf{E} = \{2, 4, 6, \dots\}$, where the notation $\dots$ should be read as *etcetera*. When we place an element after the dots as in $K = \{2, 4, 6, \dots, 100\}$, this indicates that we are talking about the finite set of even numbers greater than 1 and less than or equal to 100 (the last element on the list is 100).

Set-Builder Notation A description of the set D above may also be written using set-builder notation:

$$D = \{x |\ x \text{is an integer between 5 and 7 inclusive}\}$$

or

$$D = \{x |\ x \text{is an integer and } \ 5 \le x \le 7\}.$$

Here the symbol | is read as "such that" and the upper mathematical sentence above reads as " D is equal to the set of all x such that x is an integer between 5 and 7 inclusive".

The Empty Set The empty set is the set with no elements, i.e. the list of its elements is a blank list. It is denoted by the symbol $\emptyset$. One can think of the empty set as an empty list: { }. As you can imagine, this set can have many verbal descriptions; for example;

$\{$all major league baseball players who got more than 80 home runs in a single season$\} = \emptyset$.

Subsets: A **Subset** of a set A is a subcollection of elements of A We have B is a subset of A, denoted by $B \subseteq A$, if every element of B is also an element of A. We say that B is a **proper subset** of A if $B \subseteq A$, but $B \neq A$.

Examle If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$, $C = \{3, 4, 5, 6\}$ and $D = \{2, 6\}$,

Is $D \subseteq A$?

Is $D \subseteq B$?

Is $D \subseteq C$?

Is D a proper subset of B?

Note A itself is a subset of A, because it complies with the requirement in the definition above. Also $\emptyset \subseteq A$ for every set A.

Universal set Sometimes we need to restrict our attention to a particular set, called the universal set and usually denoted by U.

Example If we wish to do a survey on the music preferences in our class, we are restricting our attention to the class. In this case our universal set is $U = \{$all students in our class$\}$.

If we let $R = \{$students in the class who like Rap music$\}$,

$C = \{$students in the class who like Classical music$\}$,

and $E = \{$students in the class who like 80's music$\}$,

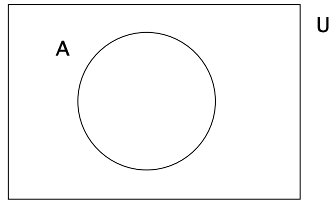
then R, C and E are all subsets of our universal set U.
Note To avoid ambiguity in the definition of such sets, it is common in surveys such as this to restrict answers to the given questions to "yes" and "no"; (e.g. "Do you like Rap music? Yes No (circle one)"). In doing this the resulting sets are well defined, but of course we have ignored the tastes of those who like some rap music but not all.

Complement of a set The complement of a set is defined with respect to a universal set, this puts a boundary on its size. If A is a subset of a universal set U, then A' or A^c (both notations are used in the homework), read A complement, is the set of all elements of U, which are **not** in A.

Example (*) If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{2, 4, 6, 8\}$, what is A'?

Example Give a verbal description of R^c in the class survey example above.

Venn Diagrams We can represent a set A in a universal set U by a picture, as follows:



The region inside the loop marked A represents the set A and the region outside it is A'.

Union If A and B are sets , Then $A \cup B$, read A union B, is a new set. Its elements are those objects which are in A **or** in B or in both i.e. those elements which are in at least one of the sets.
[In the english language, the word "or" is ambiguous. it may be an **inclusive** "or" as in : "To check in for my domestic flight, I need a current drivers license **or** a current passport", meaning that I need **at least one** of the things listed. On the other hand, the "or" may be **exclusive** as in "I can wear my cowboy boots or my Uggs to dinner today", meaning that I can wear **at most one** of the two options. **The "or" in the definition of $A \cup B$ corresponds to the inclusive "or"** and it is superfluous to add "or in both" to the definition.]

Example If $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6, 8\}$, list the elements of the set $A \cup B$.

Example Give a verbal description of the set $R \cup C$ from our class survey example. Are you in this set?

Union of 3 sets If A and B and C are sets, their union $A \cup B \cup C$ is the set whose elements are those objects which appear in at least one of A **or** B **or** C.

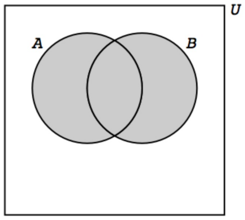
Example 2 If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$, list the elements of the set $A \cup B \cup C$.

Example If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$ are subsets of the universal set $U = \{1, 2, 3, \ldots, 10\}$, list the elements of the set $A^c \cup B \cup C$.

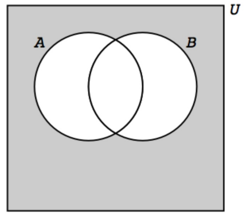
Example Give a verbal description of the set $R \cup E \cup C$ from our class survey example. Are you in this set?

Give a verbal description of the set $(R \cup E \cup C)' = (R \cup E \cup C)^c$. Are you in this set?

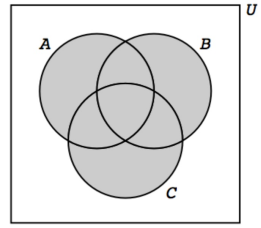
Venn Diagrams We can also draw representations of two and three subsets of a universal set using Venn diagrams as shown below. The shaded regions below represent the given subsets of the universal set. (Note that in some cases, there are two set theoretic descriptions of the same set.)



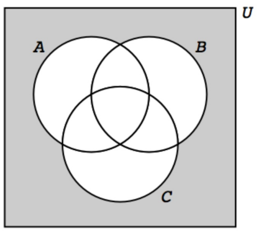
$A \cup B$
$(A' \cap B')'$



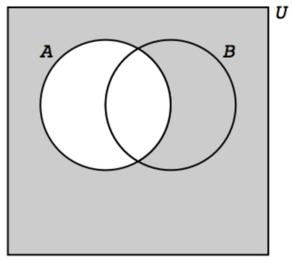
$A' \cap B'$
$(A \cup B)'$



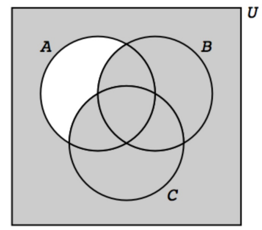
$A \cup B \cup C$
$(A' \cap B' \cap C')'$



$A' \cap B' \cap C'$
$(A \cup B \cup C)'$



A'



$A' \cup B \cup C$
$(A \cap B' \cap C')'$

Note To find the shaded region corresponding to the union of two sets on a Venn diagram, one can shade both sets individually and the the resulting shaded region corresponds to the union of the two sets.

The following interactive Venn diagram applet on Wolfram Alpha will allow you to experiment with identifying shaded regions of Venn diagrams:

Intersection If A and B are sets , Then $A \cap B$, read A intersection B, is a new set. Its elements are those objects which are in A **and** in B i.e. those elements which are in both sets.

Example If $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6, 8\}$, list the elements of the set $A \cap B$.

If A and B and C are sets, their intersection $A \cap B \cap C$ is the set whose elements are those objects which appear in A **and** B **and** C i.e. those elements appearing in all three sets.

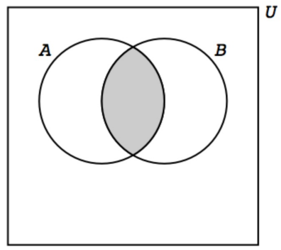
Example If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$. List the elements of the set $A \cap B \cap C$.

Example If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$ are subsets of the universal set $U = \{1, 2, 3, \ldots, 10\}$, list the elements of the set $A' \cup (B \cap C)$.

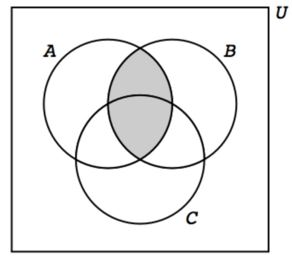
Example Give a verbal description of the people in $R \cap E \cap C$, where R, E and C are the sets described in our class survey example. Are you in this set?

Give a verbal description of those in the set $R' \cap E$. Are you in this set?

The following **Venn diagrams** show shaded regions corresponding to some intersections. In general, to find the shaded region corresponding to two sets A and B, you should shaded the sets A and B in different colors and the set $A \cap B$ will be the region where both shadings(colors) occur.



$A \cap B$
$(A' \cup B')'$



$A \cap B$
$(A' \cup B')'$

For representations of more intersections open the applet .

Disjoint sets Two sets A and B are said to be disjoint if $A \cap B = \emptyset$. For example if A = {All major league baseball players who got more than 700 home runs in their career} = { Barry Bonds, Hank Aaron, Babe Ruth} and B = {All major league baseball players with a career batting average greater than .350} = { Ty Cobb, Rogers Hornsby, Joe Jackson}, then A and B are disjoint, i.e. $A \cap B = \emptyset$.

Note The empty set has the following properties: For any set A,

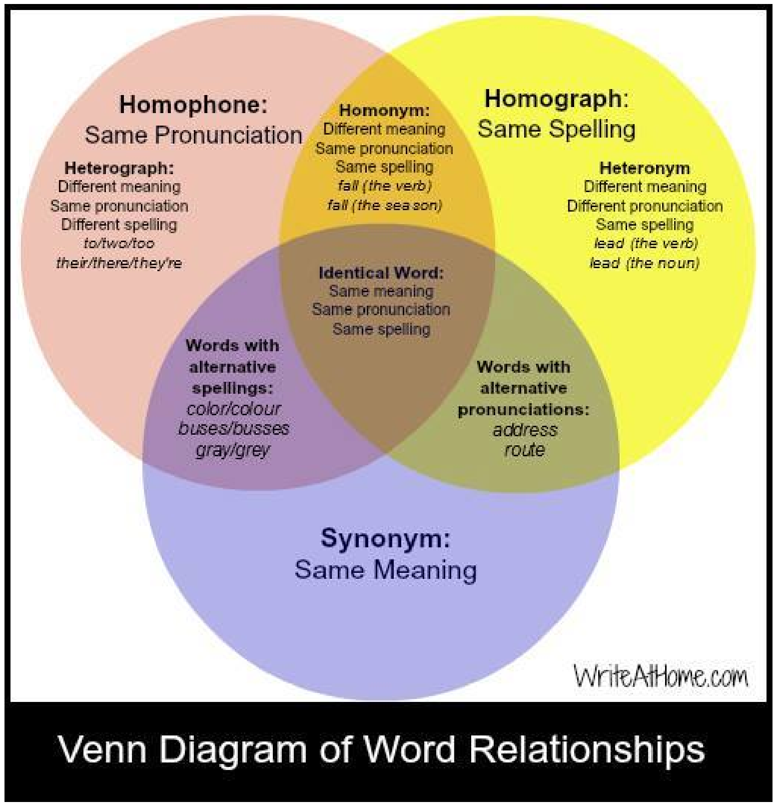
$$\emptyset \cup A = A, \quad \emptyset \cap A = \emptyset \quad \text{and} \quad \emptyset \subset A.$$

Note Complements have the following properties:

$$A \cap A' = \emptyset, \quad (A')' = A \quad A \cup A' = U$$

Verify these properties for example (*) above.

Venn diagrams of two or three sets are often used in presentations. Venn diagrams of more sets are possible, but tend to be confusing as a presentation tool because of the number of possible interactions.



The following diagrams show Venn diagrams for five sets on the left and for 7 sets on the right.